

Dynamic analysis of a free piston Stirling engine working with closed and open thermodynamic cycles

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ABSTRACT

In free piston Stirling engines the power generated by the engine is related to the length of the piston and displacer strokes. Length of strokes vary with respect to the hot end temperature, external load, charge pressure, rod diameter, stiffness of springs, masses of piston and displacer and static positions of the piston and displacer. When the length of displacer and piston strokes exceeds the estimated limits, some mechanical collisions occur between piston and displacer or displacer and cylinder. In this work, the dynamic model of a free piston Stirling engine working with closed and open thermodynamic cycles was derived and numerically solved for an optional pair of the piston and displacer masses. Safe ranges were investigated for the hot end temperature, charge pressure, damping coefficient of the piston motion, stiffness of the piston spring and area of the displacer rod. The stiffness of the displacer spring and static positions of the piston and displacer were optimized. Analysis indicated that, a free piston Stirling engine working with a closed thermodynamic cycle performs a stable operation within a small range of the hot end temperature and damping coefficient of the piston motion. By means of inverting the engine into an open-cycle engine, the limited range of the hot end temperature and the damping coefficient of the piston motion were partially enlarged.

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1. Introduction

The free piston Stirling engine is a simple mechanism generating a translational motion in the form of a vibration. It was invented by Beale in 1964 [1,2]. The free piston Stirling engines consist of a few number of components such as piston, displacer, springs and casing. The machine works with any kind of heat source such as solar energy, radioisotope energy, fossil fuels, geothermal energy, damp heat, wood, coal etc [3–5]. Despite that the free piston Stirling engines have a large potential of use, solar energy and micro cogeneration are the most plausible two of its use area [6,7].

In free piston Stirling engines, no leaking problem exists because of that the engine is built hermetically. The work generated by the engine may be extracted in the form of the mechanical energy, fluid power and electricity. For the fluid power and mechanical energy, the casing motion of the engine may be utilized and this type of engine is sometimes named as free cylinder Stirling engine which has a lighter cylinder than its piston [8]. The electricity may be generated via the magnetic interaction between the piston and a magnetic field induced by a coil that is wound around the engine's cylinder externally.

The most important priority of the free piston Stirling engine is that when the hot end of its cylinder is heated, it runs without any external excitation [9]. Due to the absence of the side forces exerting on the piston, the displacer and the cylinder of the free piston Stirling engines, its lifetime is long enough. Inherently, absence of side force reduces the power dissipation caused by frictions as well [10].

In free piston Stirling engines the strokes of piston and displacer, working volume pressure, phase difference between piston and displacer, masses of the piston and displacer, the mass of casing, the flow friction of working fluid, the load applied by the electricity generator, the rod diameter etc are parameters affecting the performance of the engine and interrelated among them. Differently from the kinematic engines, the working performance a free piston Stirling engine may be predicted via a dynamic modelling instead of a thermodynamic modelling.

It is seen that the first analysis conducted on the dynamic behaviour of the free piston Stirling engines was performed by Beale [2]. The studies conducted by Redlich and Berchowitz [11] and Benvenuto [10] treated the free piston Stirling engines as a dynamic system having two degree of freedom by assuming the casing to be constant. In these studies the solution of the dynamic model was accomplished analytically by means of converting into the linear form. The engines whose power is extracted through the

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Nomenclature

A_k	The crosscut area of the displacer rod (m^2)	R	The working gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)
A_p	The crosscut area of the cylinder (m^2)	t	Time (s)
C_d	The damping coefficient of the displacer motion (N s m^{-1})	T_b	The temperature of the engine block (K)
C_p	The damping coefficient of the piston motion (N s m^{-1})	T_c	The cold volume temperature (K)
h_d	The displacer length (m)	T_h	The hot volume temperature (K)
k_p	The stiffness of the piston spring (N m^{-1})	T_r	The mean temperature of the regenerative channel (K)
k_d	The stiffness of the displacer spring (N m^{-1})	V_b	The volume of the engine block (m^3)
m_d	The mass of the displacer (kg)	V_c	The volume of the cold space (m^3)
m_p	The mass of the piston (kg)	V_h	The volume of the hot space (m^3)
m_b	The mass of the gas in the block (kg)	V_r	The volume of the regenerative channel (m^3)
m_t	The total mass of the gas in the engine (kg)	x	The distance indicating the position of the piston top (m)
m_w	The mass of the gas in the working volume (kg)	x_e	The static position of the piston top (m)
p_b	The block pressure (Pa)	y	The distance indicating the position of the displacer bottom (m)
p_w	The working volume pressure (Pa)	y_e	The static position of the displacer bottom (m)

casing may be designed by means of a dynamic modelling having three degree of freedom. Rogdakis et al. [12], presented a dynamic model possessing three degree of freedom and analytically solved by means of obtaining the linear form of equations. Boucher et al. [13] carried out the analysis of a free piston Stirling engine possessing a piston and two displacers as moving component and predicted design parameters for 1 kW power output. Riofrio et al. [14] presented a dynamic model relevant to the free piston Stirling engine: Sunpower B-10B. In this engine the piston and displacer are connected to the casing of the engine via springs.

This work aimed to study the dynamic behaviours of a free piston Stirling engine whose piston and displacer were connected to the casing of the engine via springs. The power generated by the engine was assumed to be extracted through its piston. The casing of the engine was assumed to be stagnant. The dynamic model was solved numerically without linearization.

2. Mechanism and mathematical modelling

2.1. Mechanism

The mechanism and some of the used nomenclature is seen in Fig. 1. The engine consists of a displacer, a piston, two springs and the casing of the engine. The piston and displacer are connected to the casing of the engine via springs. Between the piston and displacer there is not a mechanical connection. Synchronous motion of them is accomplished by means of the gas pressure in the working volume of the engine. The mass of displacer is chosen as small as possible.

The inner volume of the engine consists of the block volume and the working volume. The working volume in itself consists of the

cold, regenerative and hot volumes. The volume named as the regenerative volume, in virtue, is the volume of the flow passage through which the working fluid performs displacement between the hot and cold volumes. Heating, regeneration and cooling processes are accomplished in this channel. In Fig. 1, the regenerative volume is presented by the gap between displacer's side surface and cylinder wall.

If the hot end of the cylinder is heated up to a certain degree of temperature, the pressure in the working volume of the engine increases proportional to the ratio of temperature increase and moves the piston and displacer downwards. Due to its smaller mass, the displacer gains a higher speed before the piston and removes the working fluid from the cold volume to the hot volume. This process results in a rather high pressure in the working volume and continues to move the displacer and piston downward. While the displacer is going away from its equilibrium point, the spring force exerting on it prevails the pressure force and decelerates it. When deceleration started, the speed of the displacer is at the highest level. Therefore, the working volume pressure is still in increasing period and continues to accelerate the piston downwards. After a while, the spring force exerting on the piston prevails the pressure force and deceleration of the piston starts. After this, the displacer returns back and starts to remove the working fluid from the hot volume to the cold volume. Due to its high speed, the piston still continues to move downward. After a while, the pressure in the working volume tends to decline due to downward motion of the piston. When the displacer gained an adequate upward velocity, decrease of working fluid pressure becomes steeper due to removal of working fluid from hot volume to the cold volume. After decreasing of the

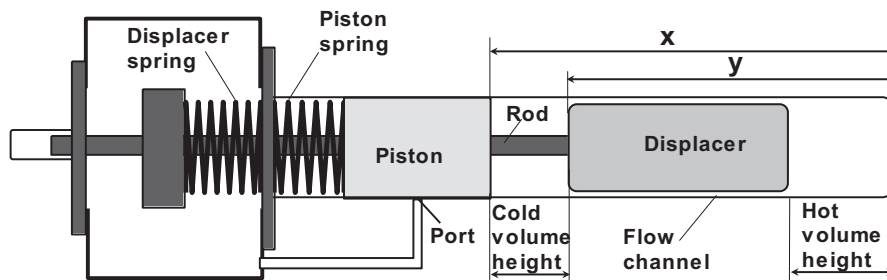


Fig. 1. Mechanism and nomenclature.

working volume pressure to about block pressure, the piston turns back and starts to compress the working fluid. The compression process continues until the end of displacer's upwards stroke. The cycle terminates when the displacer returned downwards.

To invert the closed-cycle engine into an open-cycle engine, the working volume of the engine is connected to the block volume via a channel. The end of the channel jointed to the working volume is a port which is controlled by the side surface of the piston. The port is situated near to the lower dead centre of the piston stroke. The appropriate disclosing time of the port should be about the upward returning time of the displacer. When the port is disclosed, the high pressure fluid taking part in the hot volume of the engine blows out of the working volume in the form of a pulse. The outgoing fluid passes through the regenerator and cold volume. Since it flows through the regenerator, the outgoing fluid performs a heat exchange in regenerator before exit. Therefore, we may expect a thermal efficiency near enough to the closed-cycle engine.

2.2. Mathematical modelling

Mathematical model was derived according to the assumptions:

1. The block of the engine is stagnant.
2. The temperature of the walls surrounding the working volume is not varying with time.
3. The gas temperatures in hot, regenerator and cold volumes are in thermal equilibrium with the surrounding solid walls.
4. The masses of springs are disregarded.
5. The springs are assumed to be linear.
6. The temperature of the mass taking part in the block is equivalent to the cold end temperature.
7. The leakages through the clearances between the mechanical components are disregarded.
8. The effect of the gravity is not included in the analysis.
9. The working fluid is a perfect gas.
10. All of the forces caused by dry frictions are disregarded.
11. The regenerative volume's temperature is assumed to be the average of the cold and hot volume temperatures.
12. The block pressure is variable.
13. In static equilibrium the pressure of the block and working volumes is equal.

According to the coordinates illustrated in Fig. 1, the hot and cold volumes are calculated as:

$$V_h = A_p(y - h_d) \quad (1)$$

$$V_c = (A_p - A_k)(x - y) \quad (2)$$

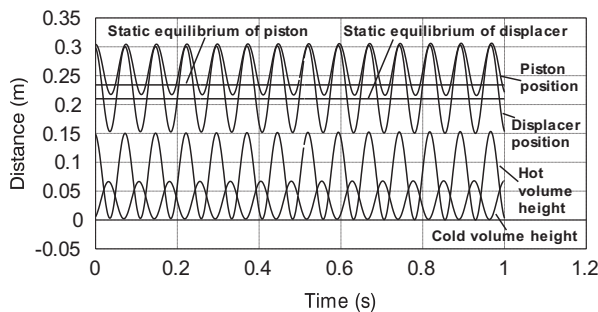


Fig. 2. Strokes of piston and displacer for 563 W of power.

Table 1
Specific values used in the case study.

Gas constant of working fluid	2077 J/kg K
Cold end temperature	300 K
Hot end temperature	700 K
Temperature of heating, regeneration and cooling channel	500 K
Volume of the heating, regeneration and cooling channel	40 cc
Area of cylinder crosscut	20 cm ²
Rod area	3 cm ²
Stiffness of piston spring	34832 N/m
Stiffness of displacer spring	10000 N/m
Damping coefficient of piston motion	80 Ns/m
Damping coefficient of displacer motion	10 Ns/m
Mass of piston	6 kg
Mass of displacer	1.5 kg
Hot volume height at static equilibrium	60 mm
Cold volume height at static equilibrium	24 mm
Total gas mass in the engine	25 g
Block volume	10 liter
Charge pressure	1525262 Pa
Displacer length (h_d)	150 mm
Static value of the cold volume height	24 mm
Static value of the hot volume height	60 mm
Location of the port in terms of x coordinate	264 mm

If the engine is assumed to work with a closed cycle, the mass of working fluid within the working volume is constant. For this case, the pressure of the working volume is:

$$p_w = m_w R \left(\frac{V_c}{T_c} + \frac{V_h}{T_h} + \frac{V_r}{T_r} \right)^{-1} \quad (3)$$

If the engine is assumed to work with an open-cycle however, the working fluid pressure requires a two stage calculation process. For the period through which the port is closed, eq. (3) is used. For the period through which the port is disclosed, the equation:

$$p_w = m_t R \left(\frac{V_c}{T_c} + \frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_b}{T_b} \right)^{-1} \quad (4)$$

is used.

The momentum equation of the displacer was derived by regarding working volume pressure, block pressure, spring force and the combination of damping forces. The combination of damping force involves the force caused by the displacement of the working fluid between the hot and cold volumes and the lubrication resistance. The momentum equation of the displacer is:

$$\frac{d^2 y}{dt^2} = -\frac{C_d}{m_d} \frac{dy}{dt} - \frac{k_d}{m_d} (y - y_e) + \frac{A_k}{m_d} (p_w - p_b) \quad (5)$$

The momentum equation of the piston was also derived by regarding working volume pressure, block pressure, spring force and the combination of damping forces. The combination of damping force exerted on the piston is consisted of an external load

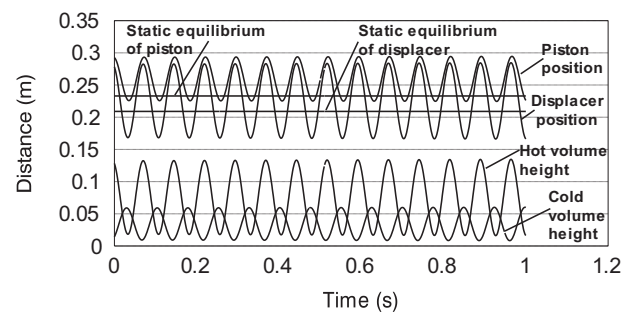


Fig. 3. Strokes of piston and displacer for 337 W of power.

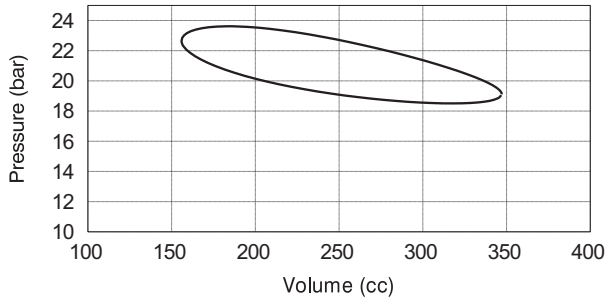


Fig. 4. P-V diagram of the engine at the optimum power.

such as electricity generation or fluid pumping and lubrication resistance etc. The momentum equation of the piston is:

$$\frac{d^2x}{dt^2} = -\frac{C_p}{m_p} \frac{dx}{dt} - \frac{k_p}{m_p}(x - x_e) + \frac{(A_p - A_k)}{m_p}(p_w - p_b) \quad (6)$$

These coupled equations were treated as an initial value problem. The initial conditions used were:

$$x = x_e, \quad \frac{dx}{dt} = 0 \quad \text{at } t = 0 \quad (7)$$

$$y = y_e, \quad \frac{dy}{dt} = 0 \quad \text{at } t = 0 \quad (8)$$

The solution of the dynamic model was accomplished numerically and investigation of the desired parameters was performed by trial and error.

3. Results and discussion

This simulation indicates that the pressure difference caused by the temperature that applied to the hot end of the engine enables it to run without any external excitation. In practice however this may be not possible due to that increase of the temperature of the hot end occurs within a time interval. During this time interval the piston and displacer move with a small enough acceleration and a new equilibrium position may occur at somewhere below the initial equilibrium point.

Fig. 2 illustrates displacements of the piston top, displacer bottom, height of the cold volume and height of the hot volume. The negative values of the hot and cold volumes indicate collisions. As seen in Fig. 2, the minimum values of both the hot and cold volume heights are set to zero to obtain the maximum power. The critical length of the displacer and piston strokes providing the maximum power were determined as 153 and 90.5 mm respectively by trial and error via changing inputs. The numerical values used in Fig. 2 were obtained using the inputs given in Table 1. Under

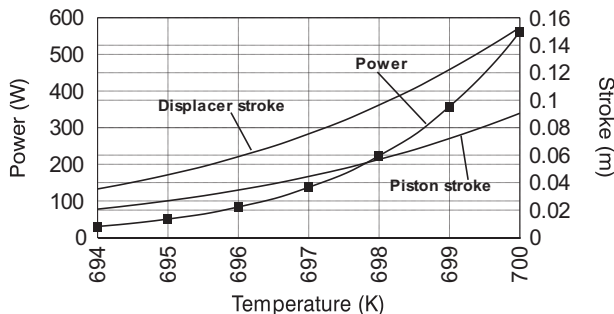


Fig. 5. Variation of the power with the hot end temperature.

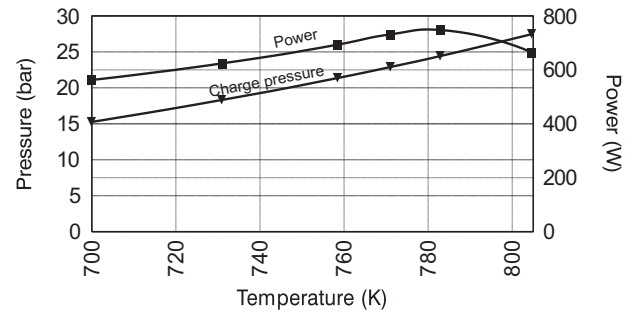


Fig. 6. Variations of the power and charge pressure with the hot end temperature.

these conditions the engine provides 563 W of power. The power was very sensitive to the static equilibrium values of the hot and cold volume heights. Small variations in these volumes may cause a drastic variation in the power. For instance, reducing the static equilibrium value of the hot volume height from 60 mm–59 mm, decreases the power from 563 W to 337 W. Corresponding to this decrement of the hot volume height, the piston and displacer strokes exhibit a significant amount of decrease as shown in Fig. 3.

Some special features of the engine such as compression ratio, pressure range and swept volume are seen on the P-V diagram of the engine, Fig. 4. The compression ratio and pressure ratio were determined as 2.2263 and 1.2684 respectively. The appearance of the P-V diagram is very similar to the one presented by Benvenuto [10].

Variation of the power, displacer stroke and piston stroke with respect to the hot end temperature are seen in Fig. 5. Due to mechanical impacts between the piston and displacer or cylinder and displacer, the hot end temperature may not be increased higher than 700 K which corresponds to the optimum power 563 W. While the other conditions were the same with that given in Table 1, if the hot end temperature is varied downwards, results seen in Fig. 5 were obtained. Corresponding to 6 K decrease of the hot end temperature, the power decreases from 563 W to 30 W. Such steep variation of power is a disadvantageous feature of this engine, since that in practice the hot end temperature may not be kept at a constant value. Due to several reasons the hot end temperature would display some fluctuations. Variations of piston and displacer strokes are also too steep and their maximum values are larger than reasonable values which should be less than 100 mm.

The power of the engine was found to be very sensitive to C_p . Only 1 Ns/m decrease of C_p causes the power to decrease from 563 W to 28 W. Since C_p is a parameter changing the load, we expect that decrease of C_p should increase the amplitude of the piston stroke. In contrast to expectation however, there occurs a decrease. The cause of this contradiction was estimated as variation of the phasing between the piston and displacer motions.

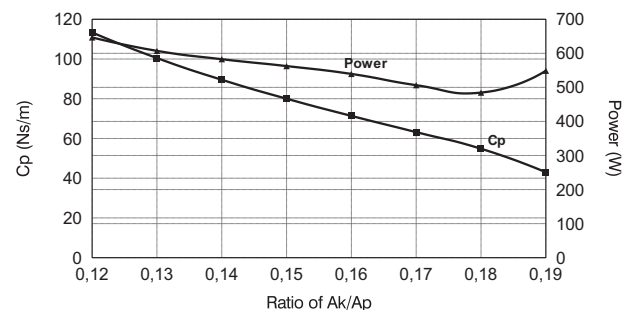


Fig. 7. Variation of the power and C_p with respect to A_k/A_p .

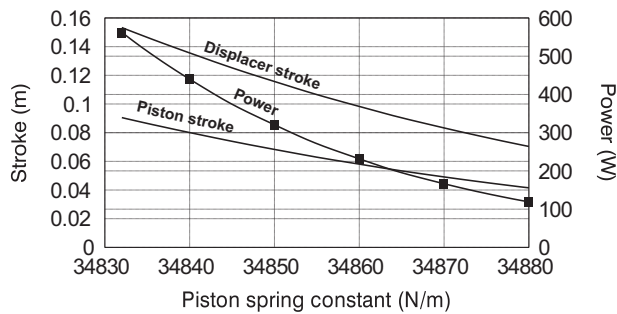


Fig. 8. Variations of piston and displacer strokes and power with respect to the stiffness of the piston spring for closed-cycle engine.

Without changing any component of the engine, its power can be increased by means of increasing the charge pressure and hot end temperature. If the charge pressure is increased while the hot end temperature is constant, the engine tends to stop. Any increase in charge pressure requires an increase in the hot end temperature. Fig. 6 indicates the relation between the hot end temperature and charge pressure which was obtained by using the inputs given in Table 1. The relation between the charge pressure and hot end temperature is almost linear. Fig. 6 also indicates the variation of the optimum power with respect to the hot end temperature or charge pressure. The power has a maximum about 780 K hot end temperature which corresponds to 24.43 bar charge pressure. Increase of the charge pressure and the hot end temperature does not change the frequency of the piston and displacer oscillations, however, causes some increase in their amplitudes whereby the power increases.

The variation of the power and damping coefficient of the piston motion with respect to A_k/A_p is illustrated in Fig. 7. To obtain the results used in Fig. 7, the value of A_k/A_p is optionally set and C_p is manipulated until obtaining maximum power which corresponds to critical values of amplitudes of displacer and piston strokes above which collision occurs. As long as A_k/A_p is lower than 0.2 the collision between components of the engine could be prevented by manipulating C_p which regulates the external load. After this, the collision becomes unavoidable. As seen in Fig. 7, decrease of A_k/A_p increases the power as well as C_p . Within the examined range of A_k/A_p , the frequency of the piston and displacer motion did not exhibit notable variations.

By using the inputs given in Table 1, when the variations of piston stroke, displacer stroke and power were examined with respect to the stiffness of the piston spring, the results illustrated in Fig. 8 were obtained. The sensitivity of strokes and power to the stiffness variation may be disadvantaging. In order to keep the physical dimensions of the engine at reasonable values, the displacer stroke should be less than 100 mm which requires $K_p > 34860$ N/m. The critical value of the piston spring stiffness is 34832 N/m below which collision appears.

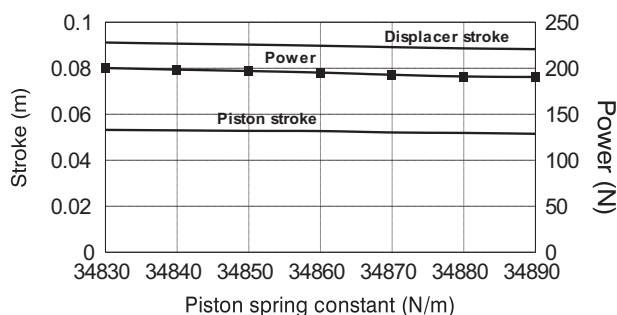


Fig. 9. Variations of piston and displacer strokes and power with respect to the stiffness of the piston spring for the open-cycle engine.

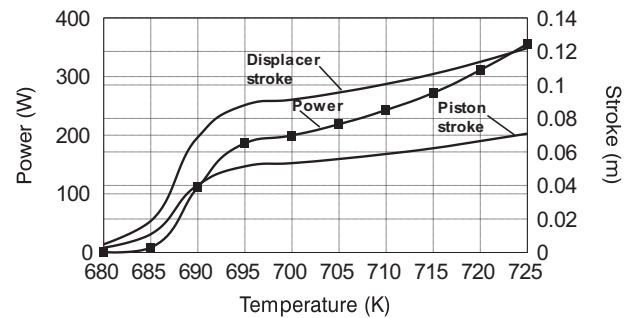


Fig. 10. The power variation of the open-cycle engine with respect to the hot end temperature.

Fig. 9 illustrates variations of the displacer and piston strokes and power with respect to the stiffness variation of the piston spring for an open-cycle engine. Results illustrated in Fig. 9 were obtained by using inputs given in Table 1. The place of the port was designed as 30 mm below the static equilibrium point of the piston top. As seen, the length of displacer and piston strokes are within the desired limits. Variations of the both strokes with respect to the constant of the piston spring are linear and very little. Related to the variation of piston and displacer strokes, the variation of power is also linear and very little.

Fig. 10 illustrates power variation of an open-cycle free piston Stirling engine with respect to the hot end temperature. Below the 690 K hot end temperature, the stroke of the piston is not adequate to disclose the port and the engine works as a closed-cycle Stirling engine. For this case the maximum power generated by the engine is 111 W. Above the hot end temperature of 690 K, the engine works as an open-cycle Stirling engine. For the hot end temperatures below 725 K, the engine works without collision. As seen in Fig. 10, while the hot end temperature varies from 695 K to 725 K, the power of the engine varies from 186 W to 355 W.

Fig. 11 shows variations of the piston and displacer strokes and power of the open-cycle engine with respect to C_p . The inputs are the same with ones given in Table 1. Within the range $78 < C_p < 83.4$ the power displays a stable variation. This range of C_p may meet the tolerance requirement of the engine.

The P-V diagram and strokes of the open-cycle engine are seen in Fig. 12 and Fig. 13 for working conditions given in Table 1. The power, piston stroke and displacer stroke appearing at 700 K hot end temperature and 80 Ns/m damping coefficient of piston motion are 200 W, 53.3 mm and 91.1 mm respectively and appropriate for practical purpose. As the result of this study a free piston Stirling engine having 50 mm piston diameter, 300 K cold end temperature, 700 K hot end temperature and 15 bar charge pressure was found to be able to generate 200 W of power.

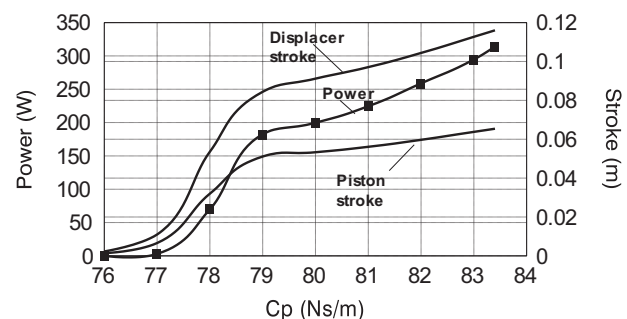


Fig. 11. Variation of the power of the open-cycle engine with C_p .

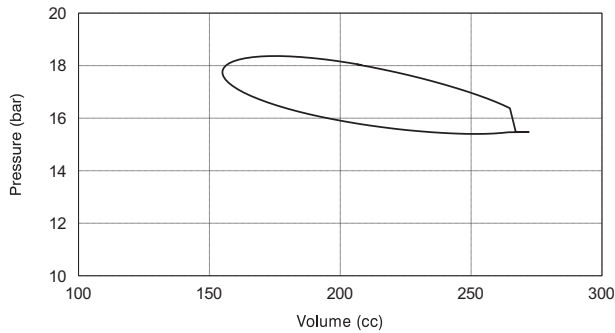


Fig. 12. P-V diagram of the open-cycle engine.

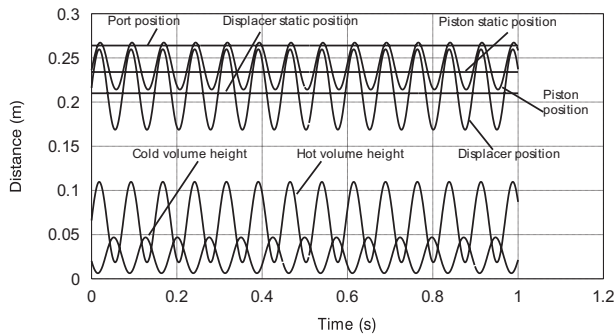


Fig. 13. Strokes of the piston and displacer and heights of cold and hot volumes of the open-cycle engine.

4. Conclusion

The dynamic model of a free piston Stirling engine was derived and numerically solved. The dynamic responses of the engine were investigated against the hot end temperature variation, the damping coefficient variation of the piston, rod diameter variation of the displacer, static positions of the piston and displacer, charge pressure variation and stiffness of the piston spring. It is determined

that the power exhibits very big variations with respect to the static position of piston and displacer, hot end temperature, piston damping coefficient, displacer rod diameter and stiffness of the piston spring. Out of a small range of these parameters either a mechanical impact appears or the engine works with a very low power. If the engine is inverted to an open-cycle engine, the factors limiting the performance of the engine are partially eliminated. As the result of this study a free piston Stirling engine having 50 mm piston diameter, 300 K cold end temperature, 700 K hot end temperature and 15 bar charge pressure was found to be capable of generating 200 W of power.

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